On the multiple steady-state fully resonant water waves

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Abstract:

The steady-state fully resonant waves in deep and finite water depth governed by exactly nonlinear wave equations are investigated analytically, using the so-called homotopy analysis method (HAM). The multiple steady-state resonant waves are obtained, with time-independent amplitudes and wave spectrum, i.e. without exchange of wave energy between different wave components. Besides, it is found that the resonant wave component may contain rather small proportion of wave energy. Similar conclusions are obtained for the so-called class-I Bragg resonance of progressive waves obliquely propagating over an undulate bed with an infinite number of sinusoidal ripples. It is found that there also exist the multiple, steady-state class-I Bragg resonant waves, whose wave spectrum is time-independent. In addition, the class-I Bragg resonant wave may contain less, equal or more wave energy than the incident wave, corresponding to the reflection coefficient R ¡ 1, R = 1 and R ¿ 1, respectively. Therefore, the multiple steady-state resonant waves exist not only in wave-wave nonlinear interaction but also in wave-bottom nonlinear interaction. All of these might deepen our understanding and enrich our knowledge about the resonance of gravity waves.

References

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